A Natural Framework for Chaotic Inflation

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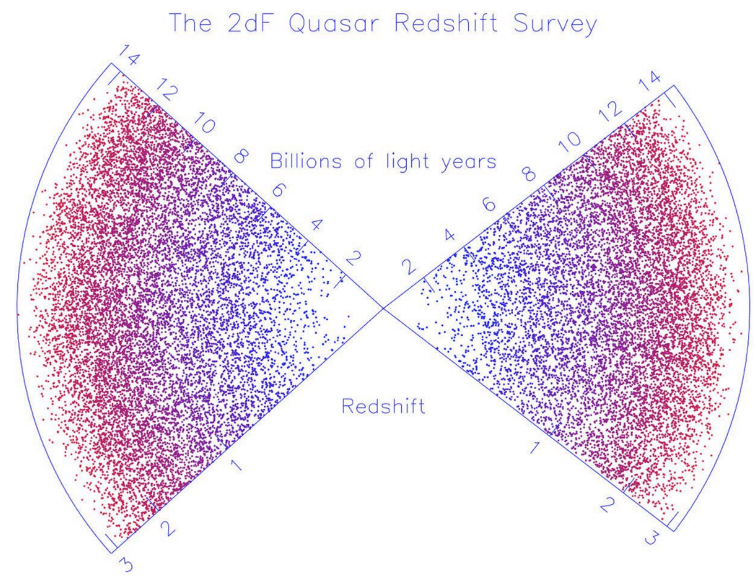
with L. Sorbo,
arXiv:0810.5346 (PRD), arXiv:0811.1989 (PRL)

Outline

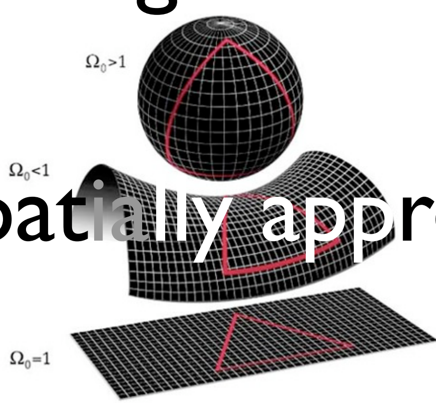
- Inflationary prolegomena
- Interactions, radiative corrections & symmetries
- 4-forms and inflation with spontaneously broken gauged shift symmetry: topological Higgs effect
- Summary

INTERESTING FACTS ABOUT THE UNIVERSE

- old and very large

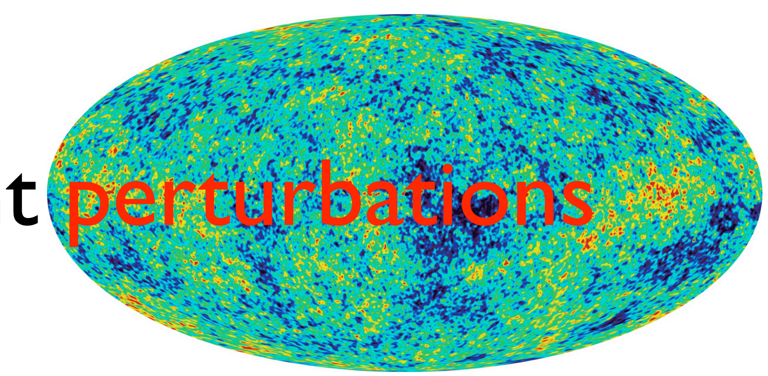


- homogeneous and isotropic

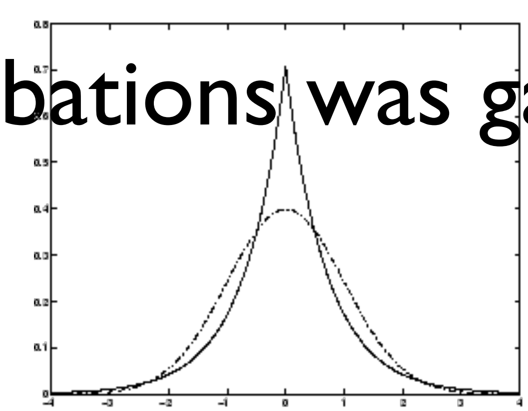


- spatially approximately flat

- structure seeded by small, scale invariant perturbations

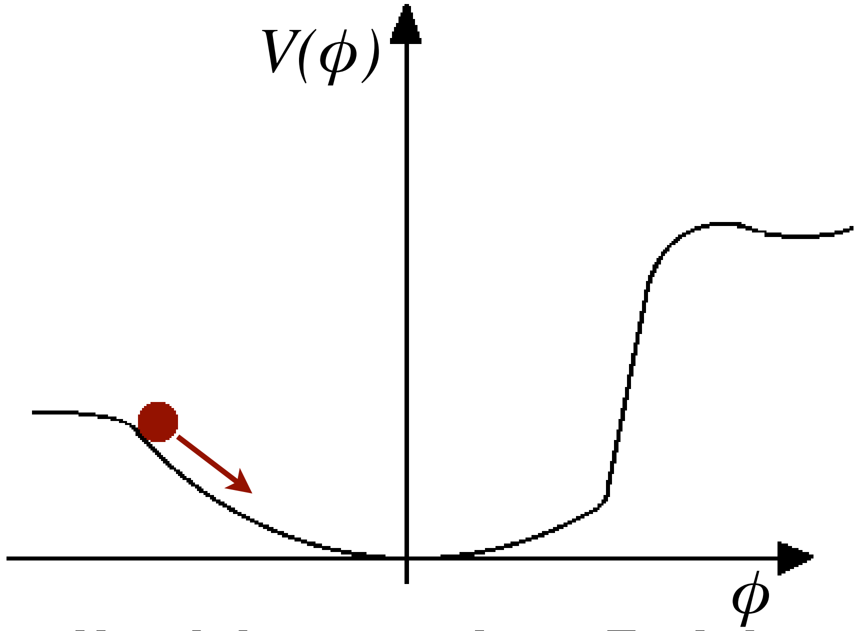


- spectrum of perturbations was gaussian



It all points to inflation...

But: what inflated?



`Standard Models': very early Universe controlled by scalar field ϕ, whose potential $V(\phi)>0$ dominates over kinetic energy

to induce acceleration, $V(\phi)$ must be *flat* $|V'(\phi)| << V(\phi)/M_P$

to have long inflation, $V(\phi)$ must *stay* flat for long enough $|V''(\phi)| << V(\phi)/M_P^2$

Single field inflation models with flat potentials are a simple way to parameterize inflationary dynamics: take a monomial potential, and allow ϕ to get large enough

The simplest example: quadratic potential

$$V(\phi)=m^2\,\phi^2\,/2$$

Amplitude of perturbations produced during inflation → $m \sim 10^{13}\ GeV$

Radiative corrections could deform the inflationary potential

Even if we write a theory with a classically flat potential for some scalar inflaton, this field cannot ignore the rest of the world: inflation must end, the universe must be repopulated (cosmological social engineering at the largest scales): the field driving inflation MUST couple to other stuff!!!

Due to quantum corrections these couplings are NOT inert: they

1- affect the functional form of $V(\phi)$

2- affect the value of the parameters that appear in $V(\phi)$

But: do they really do it?...

Oftentimes NOT! We know several explicit examples:

1) Self-interacting scalars: no, even though the daisy diagrams look dangerous: they seem to yield corrections like

$$(-1)^n \lambda^n \phi^4 (\frac{\phi}{M})^{2n-4}$$

which individually look terrible; *BUT*: they **alternate** and resum to log corrections:

$$\lambda\phi^4(1 + c\ln(\frac{\phi}{M}))$$

as in Coleman-Weinberg

2) Graviton loops: no, since they - as in induced gravity - yield finite potential and Planck mass renormalizations that go like

$$\Big(\frac{\partial_\phi^2 V}{M_{Pl}^2} + \frac{V}{M_{Pl}^4}\Big)V \qquad\qquad \partial_\phi^2 V R$$

which are small in the inflationary regime

Why? The answer is (softly broken) shift symmetry!

A *shift symmetry*: invariance under $\varphi \longrightarrow \varphi + c$; exact s.s. implies $V(\phi)$=const; this is not inflation: it needs variable $V(\phi)$ to end; so $V'(\phi)$ breaks it, but radiative corrections are proportional only to the breaking terms, going as some derivatives of $V'(\phi)$. Thus if potential is flat to start with, it will stay flat even with the corrections included, if the worst breaking comes from $V'(\phi)$.

Does it mean, there is no problem at all? NO! But: the problem is no worse than the usual radiative mass instability of a scalar which couples by relevant or marginal operators to some heavy physics - just like the Higgs mass instability.

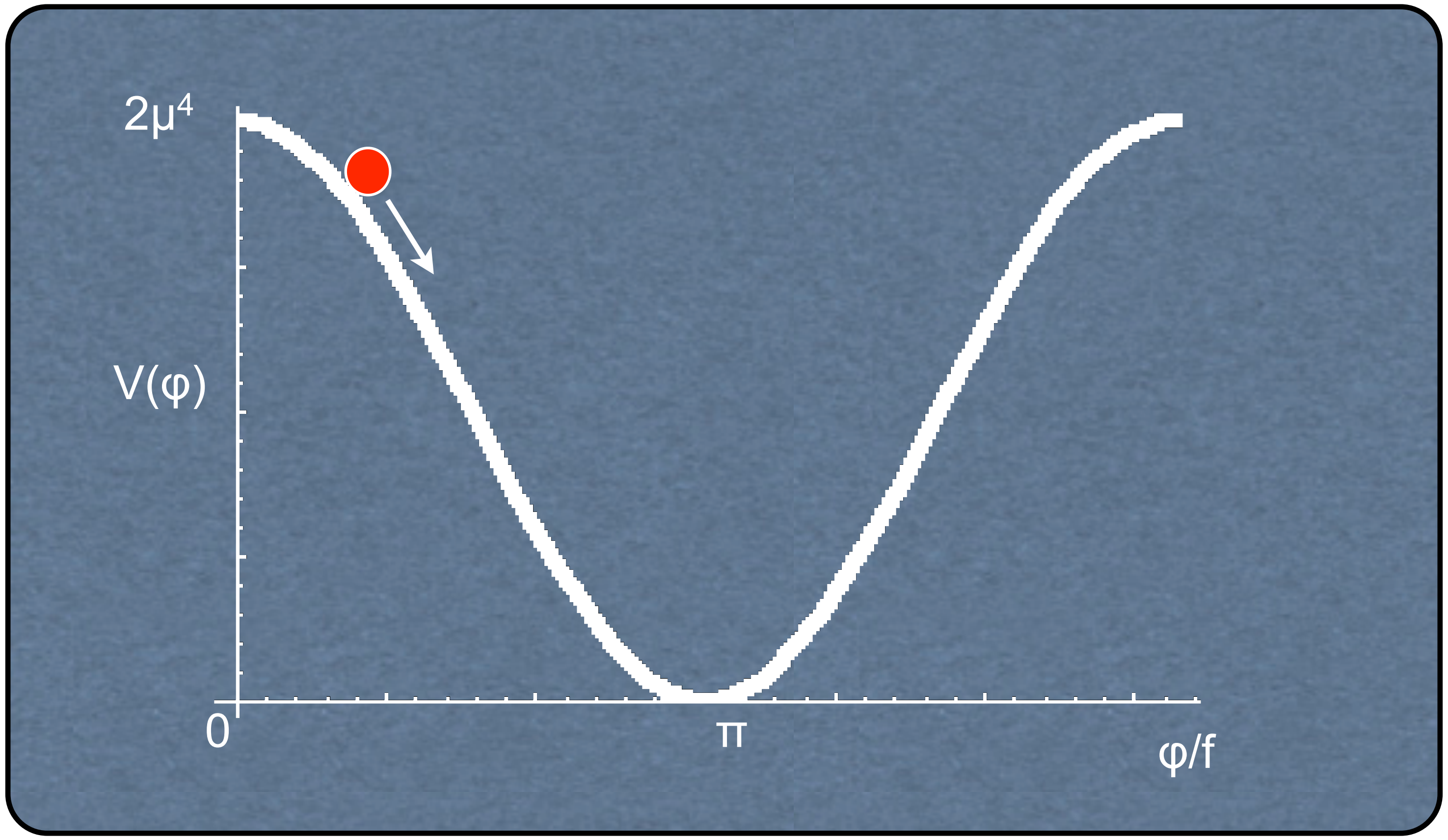
The point is, how do we generate the inflaton mass in the first place? If the mass generation mechanism evades strong shift symmetry breaking contributions, the problem will be solved!

So, can we break the shift symmetry a bit and generate a potential?

An example: using a pNGB as the inflaton

Natural inflation

$$V(\varphi)=\mu^4\,[\,\cos(\varphi/f)+1]$$



Adams, Bond, Freese, Friemann, Olinto 1990

How do we get the potential?

If ϕ is a phase, then shift symmetry ⇔ global U(1)

- Theory with a spontaneously broken global U(1)

$$\mathcal{L} = \partial_\mu H^* \, \partial^\mu H - \lambda \left(|H|^2 - v^2\right)^2$$

- Decompose $H = (v + \delta H)\, e^{i\phi/v}$

  where δH is massive and φ is a massless Goldstone boson *(pseudoscalar)*

- The global U(1) is broken by global effects e.g. gravitational instantons

$$\delta\mathcal{L} = e^{-S}\, M_P^3\, (H + H^*) + \ldots$$

(S = instanton action, ∝ M_P^n)

- A potential is generated: $$\delta V \sim e^{-S} M_P^3\, v\, \cos\left(\phi/v\right)$$

PSEUDO-NAMBU-GOLDSTONE BOSON
PNGB

To have inflation (ie get 60 or more efolds) we need

$$M_{Pl} \ll \phi \ll f$$

so, just take a very large pNGB decay constant f; easy in field theory...
However:

String Theory appears to require **f<M$_P$**

Banks, Dine, Fox and Gorbatov;
Adams, Arkani-Hamed, Motl, Vafa;

The field φ still needs to be large; this is bad, because higher harmonics in the nonperturbative potential win over the leading order term and steepen the potential... Indeed:

n-instanton actions contribute $\propto e^{-(n M_P/f)} \cos(n\,\phi/f)$ to pNGB potential

$\Downarrow$

subleading f/M_P harmonics in $V(\phi)$ matter

A different approach: use 4-forms!

$$S_{4form} = -\frac{1}{48}\int F_{\mu\nu\varrho\lambda}\, F^{\mu\nu\varrho\lambda}\, d^4x$$

$$F_{\mu\nu\varrho\lambda} = \partial_{[\mu} A_{\nu\varrho\lambda]}$$

tensor structure in 4d $\Rightarrow F_{\mu\nu\varrho\lambda} = q(x^\alpha)\, \varepsilon_{\mu\nu\varrho\lambda}$

equations of motion $D^\mu F_{\mu\nu\varrho\lambda} = 0 \Rightarrow q(x^\alpha)$ = constant

(this is why particle physicists tended to ignore 4-forms: trivial *LOCAL* dynamics)

Sources for the 4-form: membranes

$$S_{brane} \ni \frac{e}{6}\int d^3\xi \sqrt{\gamma}\, e^{abc}\, \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda A_{\mu\nu\lambda}$$

Enter the 4-form/pseudoscalar mixing...

$$\mathcal{S}_{bulk} = \int d^4x\sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots\Big)$$

Di Vecchia and Veneziano; Quevedo and Trugenberger; Dvali and Vilenkin; NK & Sorbo.

`Gibbons-Hawking' boundary terms:

$$\int d^4x\sqrt{g}\ \tfrac{1}{6}\ \nabla_\mu\big(F^{\mu\nu\lambda\sigma}A_{\nu\lambda\sigma}\big) \ - \int d^4x\sqrt{g}\ \tfrac{1}{6}\ \nabla_\mu\big(\mu\phi\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}A_{\nu\lambda\sigma}\big)$$

Action invariant under shift symmetry:

under $\phi \to \phi + c$, $\mathcal{L} \to \mathcal{L} + c\,\mu\,\varepsilon^{\mu\nu\varrho\lambda}F_{\mu\nu\varrho\lambda}/24$

What are eqs of motion?

- Direct variation of bulk action:

$$\nabla^2 \phi = \frac{\mu}{24} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma}$$

$$\nabla^2 F_{\mu\nu\lambda\sigma} = \mu^2 F_{\mu\nu\lambda\sigma}$$

- Substituting,

$$\nabla_\mu \left(\nabla^2 \phi - \mu^2 \phi \right) = 0$$

$$F_{\mu\nu\lambda\sigma} = \sqrt{g} \epsilon_{\mu\nu\lambda\sigma} \left(q + \mu\phi \right)$$

Mass

- Therefore: we have a mass term!

$$\frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma}$$

- What is UNUSUAL: this RETAINS the shift symmetry

$$\phi \to \phi + \phi_0$$

- The lagrangian changes only by a total derivative:

$$\Delta\mathcal{L} = \frac{\mu\phi_0}{24}e^{\mu\nu\lambda\sigma}F_{\mu\nu\lambda\sigma}$$

- The symmetry is broken spontaneously after a solution is picked!

Making symmetry manifest

- First order formalism: enforce F = dA with a constraint

$$\mathcal{S}_q = \int d^4x \, \frac{q}{24} \, \epsilon^{\mu\nu\lambda\sigma} \left(F_{\mu\nu\lambda\sigma} - 4\partial_\mu A_{\nu\lambda\sigma} \right)$$

NK, 1994

- Then change variables

$$\tilde{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} - \sqrt{g}\epsilon_{\mu\nu\lambda\sigma}(q + \mu\phi)$$

- This completes the square; integrate F out. What remains:

$$S_{eff} = \int d^4x \sqrt{g} \Big(\frac{M_{Pl}^2}{2} R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}(q + \mu\phi)^2 + \frac{1}{6} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} A_{\nu\lambda\sigma} \, \partial_\mu q \Big)$$

- The membrane term enforces jump on q (ie *F):

$$\Delta q|_{\vec{n}} = e$$

Mass & symmetries manifest!

- Mass term

$$V = \frac{1}{2}\left(q + \mu\phi\right)^2$$

- Shift symmetry

$$\phi \to \phi + \phi_0 \qquad q \to q - \phi_0/\mu$$

- Mass is radiatively stable; symmetry is broken spontaneously once background q is picked, as a boundary condition.
- Value of q can still change, by membrane emission

$$\Delta q|_{\vec{n}} = e$$

Note: the axion is effectively `gauging' the (discrete) shift symmetry of the non-propagating field *q;* after SSB, this field `eats' the axion; *topological Higgs effect!*

Quantization

- Classically q is continuous
- Quantum consistency requires that it be QUANTIZED! (Bousso, Polchinski)
- Example: 11D SUGRA

$$e_3 \int F_{\mu_1 \ldots \mu_4} = 2\pi n \qquad e_6 \int {}^*F_{\mu_1 \ldots \mu_7} = 2\pi n$$

- After compactification:

$$q_i = n_i \frac{2\pi M_{11}^3}{\sqrt{Z_i}} \qquad Z_e = \frac{M_{Pl}^2}{2} \qquad Z_m = \frac{M_{Pl}^2}{2 M_{11}^3 V_3^2}$$

Mass as charge

- 11D SUGRA (assume volume moduli stabilized as BP)

$$S_{11D\ forms} = M_{11}^9 \int *F \wedge F + M_{11}^9 \int A \wedge F \wedge F$$

- Truncate on $M_4 \times T^3 \times T^4$

$$A_{\mu\nu\lambda}(x^\mu) \qquad \phi = A_{abc}(x^\mu) \qquad A_{ijk}(y^i)$$

- This yields QUANTIZED MASS!

$$S_{4D forms} = -\int d^4x\sqrt{g}\Big(\frac{1}{2}(\partial\phi)^2 + \frac{1}{48}\sum_a (F^a_{\mu\nu\lambda\sigma})^2 + \frac{\mu\phi}{24}\frac{e^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma}\Big)$$

$$\mu = n\mu_0 \qquad \mu_0 = 2\pi V_3 M_{11}^3\Big(\frac{M_{11}}{M_{Pl}}\Big)^2 M_{11}$$

Avoiding instanton contributions to *V*

- Crucial for the `naturalness' of the mechanism!
  - Mass is dominated by the random 4-form fluxes
  - The instanton potential $\sim cos(\phi/f)$ coming from a gauge theory into which the axion reheats is not needed for the mass generation (actually it can spoil the potential).
  - The instanton contribution must be smaller than the 4-form one!
- Pick a ϕ which does not couple to a theory that goes strong at too high a scale; then the instantons merely yield small (and potentially interesting) bumps… like in chain inflation, or in multiple inflation.
- Similar suppression for gravitational instantons, with $f << M_P$

Corrections to our lagrangian and UV completion?

- Restricting to $F_{\alpha\beta\gamma\delta}$ and ϕ: the corrections which obey the shift symmetry and gauge invariance are powers of F^3/Λ^2 for some cutoff scale Λ: negligible as long as $\phi<\Lambda^2/\mu$

- Other moduli ψ coupled to F via terms such as $f(\psi/M)\ F^2$ in the lagrangian: their effects depend on the details of a specific string compactification

Lawrence, NK, Sorbo, Tomasiello, in progress

Connections to defect condensation in gauge theories?

- Julia & Toulouse (1979), Quevedo & Trugenberger (1996): dynamics of gauge theories with defects that condense involves a `hidden' gauge field, `revealed' by promoting a gauge field strength into a new gauge potential!

$$S_{d-h-1} = \int \frac{(-1)^h}{\Lambda^2} \Omega_{h+1} \wedge \Omega^*_{h+1} + \frac{(-1)^{h-1}}{e^2} (\omega_h - d\phi_{h-1}) \wedge (\omega_h - d\phi_{h-1})^*$$
$$+ \kappa (\omega_h - d\phi_{h-1}) \wedge T^*_h + S_M \ ,$$

- If we take # of dim to be 3, and the form rank to be 4, and take strong coupling limit ,

$$e^2 \gg 1$$

- ...this will be an effective field theory with an inflaton below the cutoff

$$m^2 \simeq \frac{\Lambda^2}{e^2} \ll \Lambda^2$$

- Such behavior is know to occur in nonlinear sigma models! (e.g. *CP(N-1)* theory in 2D, see Coleman's Erice lectures

Numerology

Note that for $M_{11}^3 V_3 \sim O(1)$: $\boldsymbol{\mu \propto M_{str} (M_{str}/M_P)^2}$

If M_{str} ~ GUT scale, and $n \sim O(1)$ then

$$\mu \sim 10^{13}\ GeV$$

as required by COBE normalization

...and by the way, wasn't ϕ an angle?

Effective potential $V(\phi)\sim(q+\mu\phi)^2$

with q, μ quantized: discrete invariance

$q \to q+n\, e,\;\; \phi \to \phi - n\, e/\mu$

Beasley and Witten 2002

at the level of action ϕ is still an angle!

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with q, μ quantized: discrete invariance

$$q \to q+n\,e, \;\; \phi \to \phi - n\,e/\mu$$

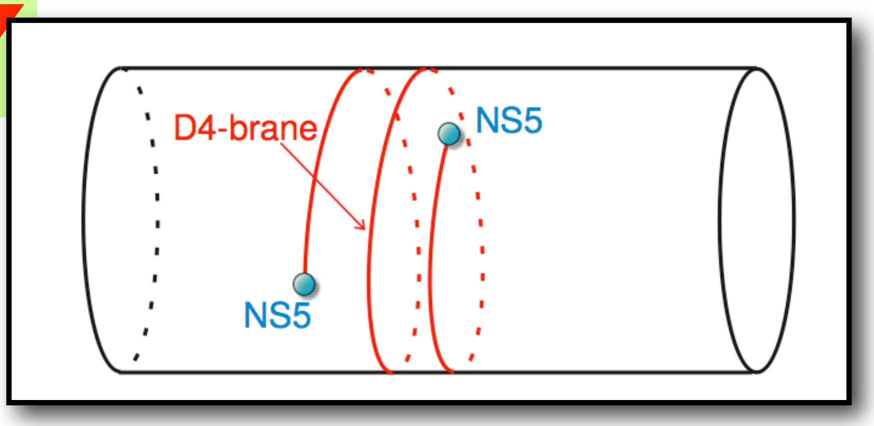
Beasley and Witten 2002

at the level of action ϕ is still an angle!

Once a vev for q is chosen, the angle unwraps:

Silverstein and Westphal 2008

MONODROMY



Signatures

- For fixed mass and 4-form charge, predictions are identical to chaotic inflation (including gravitational waves!)
- However: emission of membranes can change q (and give a kick to ϕ) during inflation
- Emission of membranes can also change μ during inflation, producing breaks in the spectrum of perturbations

PGW and the Lyth bound

r related to the inflaton displacement during inflation

(in single-field inflation)

$$\frac{\Delta\phi}{M_P} \sim \int H\, dt \sqrt{r/8}$$

and using $H\,\Delta t \sim 60$,

$$\Delta\phi \sim M_P\,(r/0.01)^{1/2}$$

observable tensor modes typically related
to a planckian excursion of inflaton

Other natural application

Quintessence in string theory

- Quintessence: (pseudo)scalar field with mass~10^{-33} eV~H_0 not yet relaxed to its minimum⇒dark energy
- $\mu \propto M_{str}\,(M_{str}/M_P)^2$ with $M_{str} > TeV \Rightarrow \mu > 10^{-20}\ eV$ too large
- Can use multiple 4-forms and multiple pseudoscalars: in type IIB SUGRA 5-form F_{ABCDE} ⇒ several 4-forms in 4d $F_{\mu\nu\varrho\lambda i}$, $i=4,\ldots,9$
- 4d action $\mathcal{S}_{eff} = \int d^4x\sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}\sum_{b=1}^{3}(\nabla\phi^b)^2 - \frac{1}{48}\sum_{a=1}^{3}(F^a_{\mu\nu\lambda\sigma})^2 + \frac{1}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\sum_{a,b=1}^{3}\mu_{ab}F^a_{\mu\nu\lambda\sigma}\phi^b\Big)$
- Elements of μ_{ac} given by fluxes of F_3 along compact dimensions
- Mass matrix $M^2{}_{ab}=\mu_{ac}\,\mu_{bc}$ can be fine-tuned to get small eigenvalues

Summary

- Naturalness of inflaton/quintessence potentials very nontrivial - but NOT impossible! One needs to formulate it carefully to see where the problems come from
- Shift symmetries: a key for constructing inflationary models
- String theory contains many 4-forms fields (used to generate the landscape of cosmological constants)
- We can use four forms to obtain radiatively stable, massive pseudoscalars with a "landscape" of masses and vevs thanks to SSB of the shift symmetry
- *Full stringy construction (as a way of proving the viability of UV complete chaotic inflation models)?*