Phy 104a, HW2

Introduction to Mathematical Physics Phy 104a

- Homework Assignment II -

Nine problems; due by: Thursday, October 20, by 5 PM at the drop-off box.

Problem 1. Page 105, Chapter 3, Section 4, Problem 21: Show that \( \vec{B} \vec{A} + \vec{A} \vec{B} \) and \( \vec{B} \vec{A} - \vec{A} \vec{B} \) are orthogonal.

Problem 2. Page 111, Chapter 3, Section 5, Problem 2: Find the slope of the line whose parametric equation is \( \vec{r} = (\vec{i} - \vec{j}) + (2\vec{i} + 3\vec{j})t \).

Problem 3. Page 124, Chapter 3, Section 6, Problem 6: The Pauli spin matrices in quantum mechanics are

\[
A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

where \( i \) is the imaginary unity, \( i^2 = -1 \). Show that \( A^2 = B^2 = C^2 = 1 \). (Note carefully that this 1 means the 2 by 2 unit matrix, and not the number 1, as is customary in quantum mechanics). Also show that any pair of matrices anticommute, that is, \( AB = -BA \) etc. Show that the commutator of \( A \) and \( B \), that is, \( AB - BA \), is \( 2iC \), and similarly for the other pairs in cyclic order.

Problem 4. Page 124, Chapter 3, Section 6, Problem 9: Use Cramer’s rule to solve the rotation equations (6.3) for \( x \) and \( y \) in terms of \( x' \) and \( y' \). Show that your results correspond to a rotation through an angle \(-\theta\).

Problem 5. Page 126, Chapter 3, Section 6, Problem 19: Find the inverse of the matrix

\[
\begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}
\]

using

a) row reduction
b) formula (6.24).
Problem 6. Page 126, Chapter 3, Section 6, Problem 22: Given the matrices

\[ A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \]

a) Find \( A^{-1} \), \( B^{-1} \), \( B^{-1}AB \) and \( B^{-1}A^{-1}B \).
b) Show that the last two matrices are inverses, that is, that their product is a unit matrix.

Problem 7. Page 129, Chapter 3, Section 7, Problem 5: Is \( \vec{F}(\vec{r}) = \vec{A} \times \vec{r} \), where \( \vec{A} \) is a given vector, a linear vector function? Prove your conclusion using (7.2).

Problem 8. Page 136, Chapter 3, Section 8, Problem 7: Solve the following set of simultaneous equations by reducing the matrix to the row echelon form:

\[
\begin{align*}
2x &- y + 3z = 1 \\
4x &- 2y - z = -3 \\
2x &- y - 4z = -4 \\
10x &- 5y - 6z = -10
\end{align*}
\]

Problem 9. Page 243, Chapter 6, Section 3, Problem 14: Prove the Jacobi identity:

\[ \vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0 \]

*Hint:* Expand each triple product as in equations (3.8) and (3.9).