

# Large Field Inflation, Ignobly

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arXiv:0811.1989 (PRL)

# Outline

- Inflation: intro
- Interactions, radiative corrections & symmetries
- Field theory picture (in progress): 4-forms and inflation with (weakly) broken (local!) shift symmetry: *a (topological) Higgs effect*
- Summary

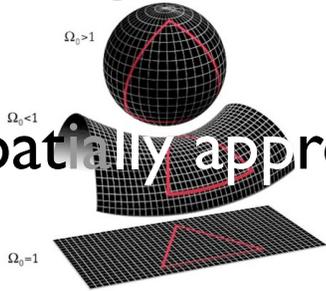
# INTERESTING FACTS ABOUT THE UNIVERSE



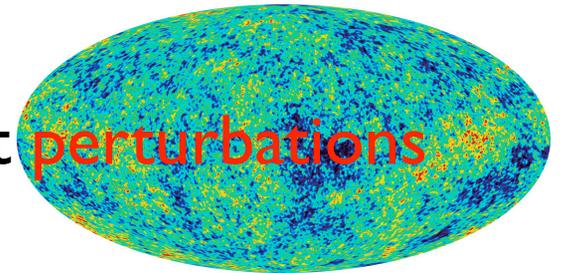
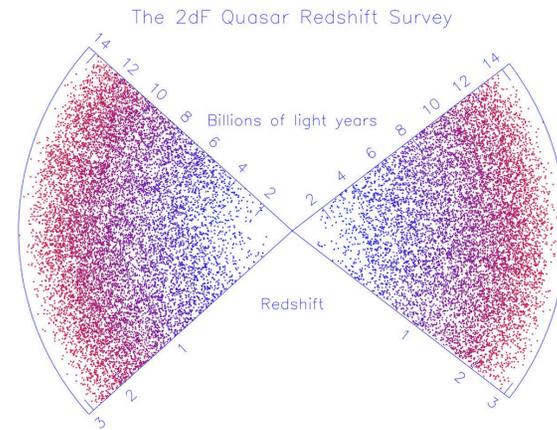
- old and very large

- homogeneous and isotropic

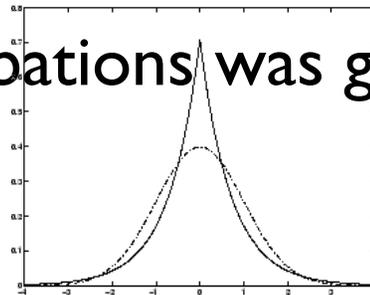
- spatially approximately flat



- structure seeded by small, scale invariant perturbations

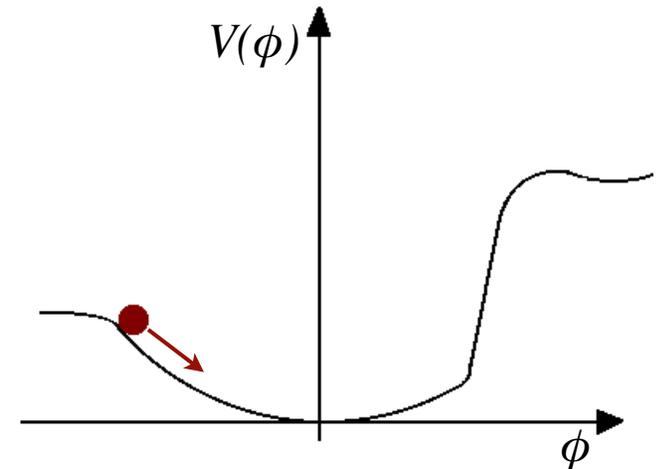


- spectrum of perturbations was gaussian



*It all points to inflation...*

*But: what inflated?*



‘Standard Models’: very early Universe controlled by scalar field  $\phi$ , whose potential  $V(\phi) > 0$  dominates over kinetic energy

to induce acceleration,  $V(\phi)$  must be *flat*

$$|V'(\phi)| \ll V(\phi)/M_P$$

to have long inflation,  $V(\phi)$  must *stay flat* for long enough

$$|V''(\phi)| \ll V(\phi)/M_P^2$$

Radiative corrections **could** deform the inflationary potential

Even if we write a theory with a classically flat potential for some scalar inflaton, this field cannot ignore the rest of the world: inflation must end, the universe must be repopulated: the field driving inflation **MUST** couple to other stuff!!!

Due to quantum corrections these couplings are **NOT** inert: they

1- affect the functional form of  $V(\phi)$

2- affect the value of the parameters that appear in  $V(\phi)$

# But: do they really do it?...

**Oftentimes NOT!** We know several explicit examples:

1) Self-interacting scalars: no, even though the daisy diagrams look dangerous: they seem to yield corrections which individually look **terrible**, like

$$(-1)^n \lambda^n \phi^4 \left(\frac{\phi}{M}\right)^{2n-4}$$

**BUT:** they **alternate** and resum to log corrections:

$$\lambda \phi^4 \left(1 + c \ln\left(\frac{\phi}{M}\right)\right)$$

as in Coleman-Weinberg

2) Graviton loops: no, since they - as in induced gravity - yield finite potential and Planck mass renormalizations that go like

$$\left(\frac{\partial_\phi^2 V}{M_{Pl}^2} + \frac{V}{M_{Pl}^4}\right) V \quad \partial_\phi^2 V R$$

which are small in the inflationary regime

## Why? The answer is a (weakly broken) shift symmetry!

A *shift symmetry*: invariance under  $\phi \rightarrow \phi + c$ ; exact s.s. implies  $V(\phi)=\text{const}$ ; this is not inflation: it needs variable  $V(\phi)$  to end; so  $V'(\phi)$  breaks it, but radiative corrections are proportional only to the breaking terms, going as some derivatives of  $V'(\phi)$ . Thus if potential is flat to start with, it will stay flat even with the corrections included, if the worst breaking comes from  $V'(\phi)$ .

*Does it mean, there is no problem at all? NO! But: the problem is no worse than the usual radiative mass instability of a scalar which couples by relevant or marginal operators to some heavy physics - just like the Higgs mass instability.*

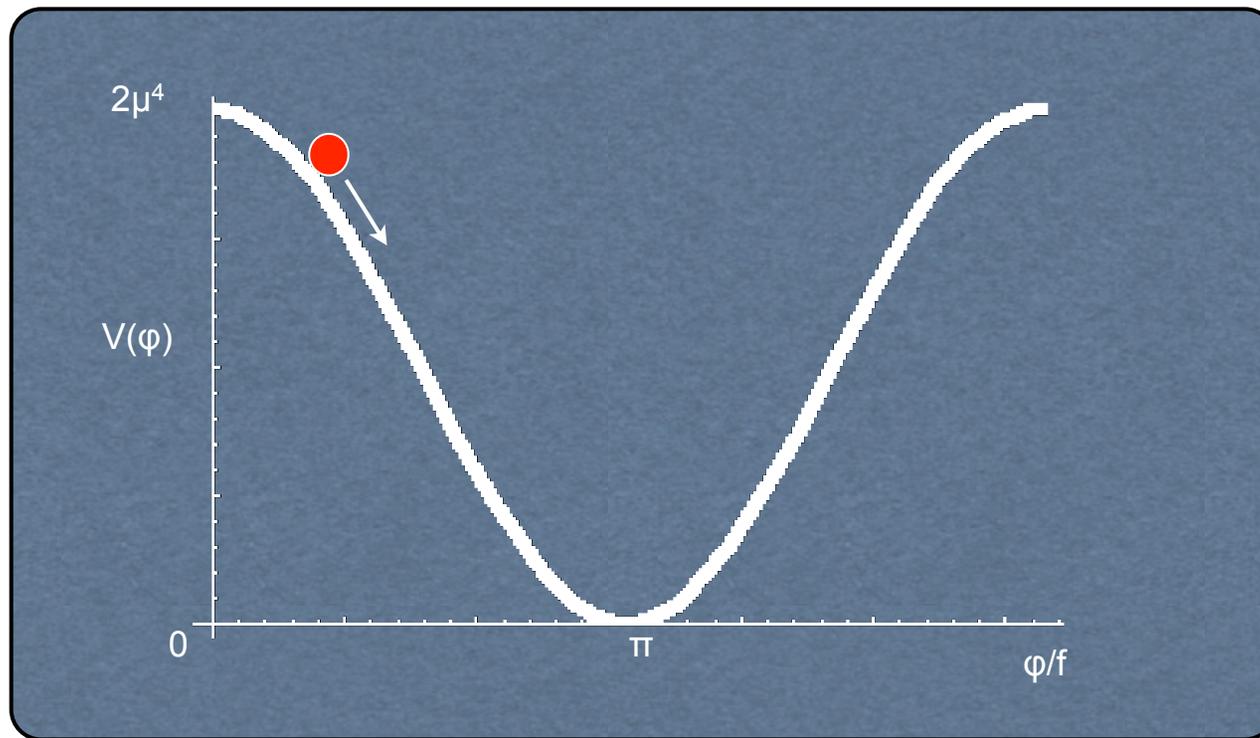
**So the point is: can we generate the mass by evading strong shift symmetry breaking?**

*The danger is that a naive shift symmetry is a fake, when gravity is turned on: a continuous shift symmetry of a simple field theory tends to be broken in quantum gravity by nonperturbative effects.*

# An case in point: a pNGB as the inflaton

Idea: natural inflation

$$V(\varphi) = \mu^4 \left( \cos(\varphi/f) + 1 \right)$$



Adams, Bond, Freese, Friemann, Olinto 1990

To have inflation (ie get 60 or more efolds) we need

$$M_{Pl} \ll \phi \ll f$$

so, just take a very large pNGB decay constant  $f$ ; easy in field theory...

However:

String Theory seems to require  **$f < M_P$**

Banks, Dine, Fox and Gorbatov;  
Adams, Arkani-Hamed, Motl, Vafa;

The field  $\phi$  still needs to be large; this is **bad**, because higher harmonics in the nonperturbative potential win over the leading order term:

$n$ -instanton actions contribute  $\propto e^{-(n M_P/f)} \cos(n \phi/f)$  to pNGB potential



subleading  $f/M_P$  harmonics in  $V(\phi)$  get big and make it wiggly

## A novel approach: use 4-forms!

$$S_{4\text{form}} = - \frac{1}{48} \int F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} d^4x$$

$$F_{\mu\nu\rho\lambda} = \partial_{[\mu} A_{\nu\rho\lambda]}$$

tensor structure in 4d  $\Rightarrow F_{\mu\nu\rho\lambda} = q(x^\alpha) \varepsilon_{\mu\nu\rho\lambda}$

equations of motion  $D^\mu F_{\mu\nu\rho\lambda} = 0 \Rightarrow q(x^\alpha) = \text{constant}$

( this is why particle physicists used to ignore 4-forms:  
trivial LOCAL dynamics )

Sources for the 4-form: membranes

$$S_{\text{brane}} \ni \frac{e}{6} \int d^3\xi \sqrt{\gamma} e^{abc} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda A_{\mu\nu\lambda}$$

## Enter the 4-form/pseudoscalar mixing...

$$\mathcal{S}_{bulk} = \int d^4x \sqrt{g} \left( \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{48} F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma} + \dots \right)$$

Di Vecchia and Veneziano; Quevedo and Trugenberger; Dvali and Vilenkin; NK & Sorbo.

'Gibbons-Hawking' boundary terms:

$$\int d^4x \sqrt{g} \frac{1}{6} \nabla_\mu (F^{\mu\nu\lambda\sigma} A_{\nu\lambda\sigma}) - \int d^4x \sqrt{g} \frac{1}{6} \nabla_\mu (\mu\phi \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} A_{\nu\lambda\sigma})$$

Action invariant under (perturbative!) shift symmetry:

$$\text{under } \phi \rightarrow \phi + c, \mathcal{L} \rightarrow \mathcal{L} + c \mu \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu\rho\lambda} / 24$$

## A simple way to see the effect of mixing

Think of it as flavor oscillations: we have the scalar propagator, 4-form propagator and scalar-form vertex:

$$\text{---} \frac{1}{p^2} \quad \text{---} \text{---} \text{---} \text{---} 1 \quad \text{---} \bullet \text{---} \mu$$

So we have a very simple sum of propagators

$$\begin{array}{c} \text{---} \\ \text{---} \bullet \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \times \text{---} \\ \text{---} \times \text{---} \times \text{---} \end{array}$$

$$\frac{1}{p^2} + \frac{1}{p^2} \mu^2 \frac{1}{p^2} + \frac{1}{p^2} \mu^2 \frac{1}{p^2} \mu^2 \frac{1}{p^2} + \dots = \frac{1}{p^2 - \mu^2}$$

This is just a **MASS TERM**

# MASS

- Therefore: we have a mass term!

$$\frac{\mu\phi}{24} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma}$$

- What is UNUSUAL: this RETAINS the shift symmetry

$$\phi \rightarrow \phi + \phi_0$$

- The lagrangian changes only by a total derivative:

$$\Delta\mathcal{L} = \frac{\mu\phi_0}{24} \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma}$$

- The symmetry is broken spontaneously after a solution is picked!

## Making symmetry manifest

- First order formalism: enforce  $F = dA$  with a constraint

$$S_q = \int d^4x \frac{q}{24} \epsilon^{\mu\nu\lambda\sigma} (F_{\mu\nu\lambda\sigma} - 4\partial_\mu A_{\nu\lambda\sigma})$$

NK, 1994

- Then change variables

$$\tilde{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} - \sqrt{g}\epsilon_{\mu\nu\lambda\sigma}(q + \mu\phi)$$

- This completes the square; integrate  $F$  out. What remains:

$$S_{eff} = \int d^4x \sqrt{g} \left( \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - \frac{1}{2} (q + \mu\phi)^2 + \frac{1}{6} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} A_{\nu\lambda\sigma} \partial_\mu q \right)$$

- The membrane term enforces jump on  $q$  (ie  $*F$ ):

$$\Delta q|_{\vec{n}} = e$$

## Mass & symmetries manifest!

- Mass term

$$V = \frac{1}{2} (q + \mu\phi)^2$$

- Shift symmetry

$$\phi \rightarrow \phi + \phi_0 \quad q \rightarrow q - \phi_0/\mu$$

- Mass is radiatively stable; symmetry is broken spontaneously once background  $q$  is picked, as a boundary condition.
- value of  $q$  can still change, by membrane emission

$$\Delta q|_{\vec{n}} = e$$

Note: the axion is effectively 'gauging' the shift symmetry of the non-propagating field  $q$ ; after SSB, this field 'eats' the axion; *topological Higgs effect! This statement is precise, which can be seen by taking the shift function to satisfy a Klein-Gordon eq!*

## Mass as charge

- 11D SUGRA (assume volume moduli stabilized as BP)

$$S_{11D \text{ forms}} = M_{11}^9 \int *F \wedge F + M_{11}^9 \int A \wedge F \wedge F$$

- Truncate on  $M_4 \times T^3 \times T^4$

$$A_{\mu\nu\lambda}(x^\mu) \quad \phi = A_{abc}(x^\mu) \quad A_{ijk}(y^i)$$

- This yields QUANTIZED MASS!

$$S_{4D \text{ forms}} = - \int d^4x \sqrt{g} \left( \frac{1}{2} (\partial\phi)^2 + \frac{1}{48} \sum_a (F_{\mu\nu\lambda\sigma}^a)^2 + \frac{\mu\phi}{24} \frac{e^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma} \right)$$

$$\mu = n\mu_0 \quad \mu_0 = 2\pi V_3 M_{11}^3 \left( \frac{M_{11}}{M_{Pl}} \right)^2 M_{11}$$

# Quantization

- Classically  $q$  is continuous
- Quantum consistency requires that it be QUANTIZED!  
(Bousso, Polchinski)
- Example: 11D SUGRA

$$e_3 \int F_{\mu_1 \dots \mu_4} = 2\pi n \quad e_6 \int {}^* F_{\mu_1 \dots \mu_7} = 2\pi n$$

- After compactification:

$$q_i = n_i \frac{2\pi M_{11}^3}{\sqrt{Z_i}} \quad Z_e = \frac{M_{Pl}^2}{2} \quad Z_m = \frac{M_{Pl}^2}{2M_{11}^3 V_3^2}$$

# Naturalness issues and shift breaking

So far we treated shift symmetry as a perturbative continuous symmetry, and that keeps radiative corrections under control.

But: in UV completions of the theory the axion is a zero mode of a form on a compact space. Its shift symmetry is a *large* gauge transformation of the field  $\phi$ . It is broken to a discrete symmetry by the periodicity of  $\phi$  (set by boundary conditions reflecting the compactness of the internal space, and given by the period  $f_\phi < M_{\text{Pl}}$ ).

This is still OK with the  $\phi$  F mixing, that remains invariant.

It still helps with non-perturbative corrections, since it implies that they will be periodic in  $f$  with the same period  $f_\phi$ .

# Breaking of discrete shift symmetry = monodromy

Silverstein and Westphal 2008

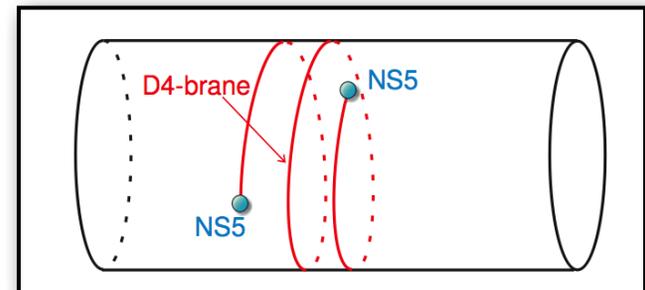
Effective potential  $V(\phi) \sim (q + \mu\phi)^2$

with  $q, \mu$  quantized: discrete invariance

$$q \rightarrow q + n e, \quad \phi \rightarrow \phi - n e / \mu \quad \text{Beasley and Witten 2002}$$

at the level of action  $\phi$  is still an angle!

Once a vev for  $q$  is chosen, the angle unwraps:



Similar ideas: Berg, Pajer and Sjors, 2009

## Instanton contributions to $V$

$$V_{inst} = \lambda_{d,inst}^4 \sum_n e^{-(n-1)c'S} \cos\left(\frac{n\phi}{f_\phi}\right) \simeq \lambda_{d,inst}^4 \sum_n \left(\frac{\lambda_{d,inst}}{\lambda_{uv,inst}}\right)^{4(n-1)} \cos\left(\frac{n\phi}{f_\phi}\right) \lll \frac{\mu^2 \phi^2}{2}$$

- Crucial for the 'naturalness' of the mechanism:
  - Mass dominated by the random 4-form fluxes. In the weak coupling, the instanton potential  $\sim \cos(\phi/f)$  coming from a gauge theory into which the axion reheats is not needed for the mass generation. With strong couplings the potential is resummed to a polynomial (Witten, LGT), but is **bounded!**
  - The instanton contribution must be smaller than the 4-form one!
- Pick a  $\phi$  so it won't couple to a theory that goes strong at too high a scale; then the instantons yield small (and potentially interesting) bumps... like in chain inflation, or in multiple inflation. This can also lead to nongaussianities through coherent amplifications (Chen, Easter & Lim; parallel talk by Flauger).
- Similar suppression for gravitational instantons, with  $f \lll M_P$

## Higher order shift-symmetry invariant terms

Gauge (and shift symmetry) invariant terms:

$$S_{\phi F} = \int d^{4+d}x \sqrt{g} \left( -F^2 + (\partial\phi)^2 + \mu\phi\epsilon F + \dots + \frac{F^{2n}}{M^{(4+d)(n-1)}} + \dots \right)$$

After dimensional reduction and stabilization of moduli:

$$\begin{aligned} S_{\phi F} &= \int d^4x \sqrt{g} \left( -V_d F^2 + V_d (\partial\phi)^2 + \mu V_d \phi \epsilon F + \dots + V_d \frac{F^{2n}}{M^{(4+d)(n-1)}} + \dots \right) \\ &= \int d^4x \sqrt{g} \left( -\tilde{F}^2 + (\partial\tilde{\phi})^2 + \mu\tilde{\phi}\epsilon\tilde{F} + \dots + \frac{\tilde{F}^{2n}}{(V_d M^{4+d})^{n-1}} + \dots \right) \end{aligned}$$

So the higher corrections are suppressed by  $V_d M^{4+d} = \Lambda_F^4 \sim M_{Pl}^4$

$$V_{eff} = \frac{\mu^2 \phi^2}{2} \left( 1 + \sum c_n \frac{\mu^2 \phi^2}{\Lambda_F^4} \right) !!!$$

## Moduli corrections

Gauge (and shift symmetry) invariant terms, after dim red:

$$\delta\mathcal{L} = c \frac{\Psi}{M_{Pl}} F^2 + \dots$$

Stabilizing the moduli with mass  $M$ , this shifts them by:

$$\delta\Psi \sim c \frac{F^2}{M^2 M_{Pl}} \sim c \frac{\mu^2 \phi^2}{M^2 M_{Pl}}$$

This corrects the scalar potential by  $M^2 \delta\Psi^2$

$$V_{corr} = V(\phi) \left( 1 + \frac{c^2}{2} \frac{V(\phi)}{M^2 M_{Pl}^2} \right) \simeq V(\phi) \left( 1 + \frac{c^2}{2} \frac{H^2}{M^2} \right)$$

OK as long as  $M > H$

# Reheating

Consider the coupling

$$\delta\mathcal{L} = \frac{\phi}{f_\phi} \text{Tr} \left( G^* G \right)$$

Reheating temperature:  $T_R \simeq \sqrt{\Gamma_{\phi GG} M_{Pl}}$

Decay rate:  $\Gamma_{\phi GG} \simeq \frac{\mu^3}{f_\phi^2}$

Thus:

$$T_R \simeq \frac{\sqrt{\mu M_{Pl}}}{f_\phi} \mu \quad \text{this is easily} \quad \gg MeV$$

So when  $f_\phi > \sqrt{\mu M_{Pl}}$  reheating temperature will be lower than the inflaton mass, and that will generically help avoid reproduction of undesired long lived relics.

## Cosmological numerology

Note that for  $M_{11}^3 V_3 \sim O(1)$ :  $\mu \propto M_{str} (M_{str}/M_P)^2$

If  $M_{str} \sim$  GUT scale, and  $n \sim O(1)$  then

$$\mu \sim 10^{13} \text{ GeV}$$

as required by COBE normalization

## PGW and the Lyth bound

$r$  related to the inflaton displacement during inflation

(in single-field inflation)

$$\frac{\Delta\phi}{M_P} \sim \int H dt \sqrt{r/8}$$

and using  $H \Delta t \sim 60$ ,

$$\Delta\phi \sim M_P (r/0.01)^{1/2}$$

observable tensor modes typically related  
to a planckian excursion of inflaton

# Signatures

- For fixed mass and 4-form charge, predictions are identical to chaotic inflation (including gravitational waves!)
- However: emission of membranes can change  $q$  (and give a kick to  $\phi$ ) during inflation
- Emission of membranes can also change  $\mu$  during inflation, producing breaks in the spectrum of perturbations
- The instanton corrections on top of the quadratic potential can also affect fluctuations, producing bumps - corrections to scale invariance and non-gaussianities (Flauger, parallel talk)

# Summary

- Naturalness of inflaton potentials very nontrivial - but NOT impossible! One needs to formulate it carefully to see where the problems come from
- Shift symmetries: a key for constructing inflationary models
- String theory contains many 4-forms fields (used to generate the landscape of cosmological constants)
- We can use four forms to obtain radiatively stable, massive pseudoscalars with a “landscape” of masses and vevs thanks to only weak breaking of shift symmetry
- *Full stringy construction (as a way of proving the viability of UV complete chaotic inflation models)?*